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Statistic analysis of the boiling curve for a droplet

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Abstract—Statistic analysis of the experimentally determined values of a water droplet's evaporation times was made. Measurements were taken from heating surface temperatures characteristic for liquid phase natural convection up to film boiling of a droplet. The results obtained confirm the hypothesis of two boiling curves in the region of transition boiling proposed by Witte and Lienhard. They also allow this concept on the region of nucleate boiling to be expanded.

INTRODUCTION

THE EXPERIMENTAL data concerning the minimum film boiling temperature are characterized by a great scatter [1], which leads to the conclusion that not just one value of this temperature but the whole range, in which the change of boiling regime is possible, should be determined. This opinion is confirmed by the results of experimental investigations done by many other investigators [2]. Michiyoshi and Makino [3] determined the whole boiling curve, natural convection → nucleate boiling → transition boiling → film boiling, for a water droplet and for four different heating surface materials. The droplet's evaporation time as a function of surface temperature, τ vs ΔT, was the quantity measured. The experimental data of these investigators are also characterized by a great scatter, especially in the region of transition boiling, and by a smaller one in the region of nucleate boiling. The smallest scatter of these results occurred in film boiling and natural convection. Witte and Lienhard [4], on the grounds of the analysis of many experimental data, came to the conclusion that in transition boiling the occurrence of two boiling curves, q vs ΔT, is possible. They distinguished the 'transition nucleate boiling' and the 'transition film boiling' curves. The first one was obtained by making the experimental measurements, q vs ΔT, from nucleate to film boiling, at increasing heating surface temperatures. The curve of 'transition film boiling' was obtained by conducting the same experiment in the direction of decreasing heating surface temperature.

The results of the measurements [1–3], mentioned above, and the considerations of Witte and Lienhard [4] prompted the authors to put forward the hypothesis that a liquid droplet's evaporation time in nucleate and transition boiling is not the quantity univocally determined by heating surface temperature. It means that a certain droplet's evaporation time at a given surface temperature can be expected only with a certain probability.

The emotional appeal of Witte and Lienhard [5] for more experimental data was one of the reasons for undertaking the experiment discussed here.

EXPERIMENTAL TECHNIQUE

In order to verify the hypothesis the experimental investigation, relying on repeated measurements of droplet evaporation time, was carried out. It means that the droplet of constant volume and constant initial temperature evaporated many times on a heating surface of constant initial temperature. The study was conducted in such a way as to eliminate thoroughly all the factors which could cause scatter of experimental data. A diagram of the experimental set-up is shown in Fig. 1. In the investigation bidistilled water, the reservoir of which was inside the protective glass shield, was used. This shield protected the water against impurity and also helped to keep its temperature constant as well as the temperature of the heating element, during the measuring cycle. The allowed fluctuation of the water temperature was ±0.1°C throughout the cycle. The water temperature in the reservoir—and so the initial droplet temperature—was equal to the ambient one and during all these measurements was 20–22°C. Constant volume of a droplet was obtained by using a laboratory micropipette of adjustable volume. Accuracy of the micropipette was ±1%, whereas repeatibility of droplet volume was from ±0.6 to ±1%. The volume of a droplet was experimentally adjusted in such a way that all water contained in the injection nozzle of the micropipette could detach from it as a single one. The volume was 1.75×10^{-8} m^3.

In order to acquire a heating surface of repeatable features, several heating elements were made of copper using the technique of extrusion with one die steel punch to form the shape of the element. In this way the identical, mirror-like heating surfaces of the shape shown in Fig. 2 were gained. The copper surface was then covered by two protective layers by cathode sputtering to protect it from oxidation. The first layer was

NOMENCLATURE

A	asymmetry coefficient, $n^{-1}\sigma^{-2}\sum_{i=1}^{j} n_i(\tau_i-\bar{\tau})^3$
E	excess, $n^{-1}\sigma^{-4}\sum_{i=1}^{j} n_i(\tau_i-\bar{\tau})^4-3$
m	number of class intervals
n	sample size
n_i	sample size of a particular class interval
q	heat flux
T	temperature
ΔT	T_p-T_{sat}
V	coefficient of variation, $\sigma\bar{\tau}^{-1}$ 100%.

Greek symbols

p	probability, $n_i n^{-1}$
σ	mean standard deviation, $\left(n^{-1}\sum_{i=1}^{j} n_i(\tau_i-\bar{\tau})^2\right)^{1/2}$
τ	droplet's evaporation time
ϕ	diameter.

Subscripts

max	maximum
min	minimum
p	heating surface (plate)
ref	reference value
reg	value on a regression line
sat	saturation.

Superscripts

$\bar{}$	average value.

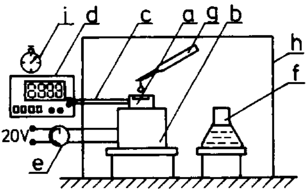

FIG. 1. Diagram of the experimental set-up for measuring droplet evaporation time: a, heating element (Fig. 2); b, heater; c, thermocouple; d, digital voltmeter; e, autotransformer; f, reservoir with distilled water; g, micropipette; h, glass protective shield.

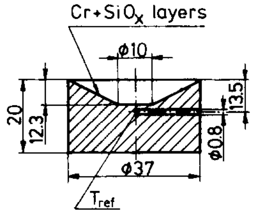

FIG. 2. Heating element.

chromium on copper, the second SiO_x on chromium. Thus the plane mirror was acquired on the bottom of the heating element, which allowed for optical control of settling impurities by a reading glass of 20 × magnification. The layer of SiO_x allowed chemical agents for cleaning the surface to be used. Analytically pure, dilute citric acid and ethanol 96% were used. If microcracks of insulating layers were observed on the bottom surface (Fig. 2), the heating element was replaced by a new one. Under the bottom of the die stamping a chromel–alumel thermocouple ($\phi = 0.5 \times 10^{-3}$ m) was inserted for determining the initial value of the reference temperature (Fig. 2), i.e. the value before a droplet was put on the heating surface. The thickness of two protective layers (Fig. 2) was not measured so the distance between the point of contact of the thermocouple and the bottom of the die stamping was unknown. For the sake of accuracy, the reference temperature, T_{ref}, was used in the description of the experimental results, instead of the surface temperature, T_p. Temperature measurements were carried out to an accuracy of ±0.001 mV. Fluctuations of the reference temperature of ±0.05 K were admitted for one measuring cycle. The cycle consisted of measuring the evaporation time of each of a hundred droplets evaporating on a heating surface at a given and fixed initial reference temperature. After evaporation of one droplet the temperature had to reach the given value and then the measurement was repeated. Water droplets were put onto the surface from a constant height of 20×10^{-3} m. The time was measured with a stop watch exact to ±0.01 s. The average duration of one measuring cycle, 100 measurements, was about 5 h.

THE RESULTS OF THE EXPERIMENTAL STUDY AND STATISTIC ANALYSIS

The measurements of the evaporation time were performed for each of 20 given values of the initial reference temperature of the heating surface, from temperature characteristic for liquid phase natural convection through nucleate and transition up to the film boiling regime of a droplet. Arithmetic mean values of the evaporation times for a particular temperature are shown in Fig. 3.

The experimental data were elaborated statistically. Because presentation of any general principles of dividing the experimental results into classes [6] is not possible, one suggestion was applied saying that the number of class intervals m should fulfill the following inequality with sample size n

$$0.5n \leqslant m \leqslant \sqrt{n}. \quad (1)$$

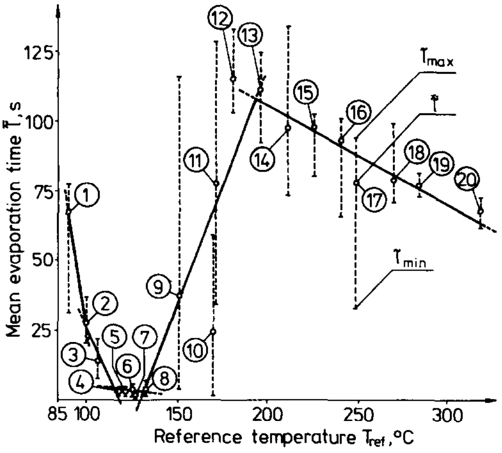

FIG. 3. The regression lines and mean arithmetic times of water droplet evaporation as a function of initial reference temperature. For each measuring point minimum and maximum values of the evaporation time are given.

The sample size of each class interval should not be less than 5. This condition was not always fulfilled as the principle of not rejecting the extreme results was accepted. Firstly, because the experiment was conducted under all conditions for obtaining repeatable results. Secondly, for fear that by eliminating the classes of sample size $n_i < 5$ from the consideration, the results obtained may be too averaged. Too large a number of classes and at the same time, very small class intervals, do not give a clear representation either, and reveal incidental deviations connected with the influence of side factors. Too small a number of classes smoothes over important details of the structure of the whole sample and in this way information included in particular data is lost. According to equation (1) evaporation times for a given initial reference temperature were divided into eight equal classes to create a distributive series, presented as histograms in Fig. 4. The bases of rectangles on the X-axis, Fig. 4, are equal to class intervals and their heights determine the sample size of these intervals.

The following quantities were calculated to carry out statistic analysis: arithmetic mean for whole sample, $\bar{\tau}$; quantities belonging to dispersion characteristics, i.e. mean standard deviation, σ, and the coefficient of variation, V; and also the coefficient of asymmetry, A, and excess, E [6]. The coefficient A defines the symmetry of a distributive series towards arithmetic mean; excess E is the measure of the deviation of the series towards the normal distribution series. All the mentioned quantities are shown on the histograms in Fig. 4.

To make the analysis easier, the dispersion characteristics are gathered together in Table 1. In accordance with Fig. 3 the following measuring points are ascribed:

(a) points ① and ② → region of liquid phase natural convection (200 measurements);

(b) points ②–④ → region of nucleate boiling (300 measurements);

(c) points ④–⑦ → region of nucleate boiling crisis (400 measurements);

(d) points ⑦–⑫ → region of transition boiling (600 measurements);

(e) points ⑫–⑳ → region of film boiling (900 measurements).

Table 1. Dispersion characteristics (see Fig. 4)

Measuring points	σ		V	
	σ_{min}	σ_{max}	V_{min}	V_{max}
①–②	3.046	8.493	10.510	12.578
②–④	0.624	3.476	10.510	27.110
④–⑦	0.252	0.983	24.479	40.690
⑦–⑫	0.252	29.128	6.000	66.227
⑫–⑳	1.824	24.686	2.351	15.480

The next step of the statistic analysis consisted in assuming the simplified forms of the relations describing evaporation time as a function of surface temperature (Fig. 3), and in determining the probability distribution of deviations of the real times towards the values assumed as systematic. In other words, in numerated earlier characteristic intervals of the reference temperature, regression lines were drawn using the library subroutine S4TGAE [7] and histograms of the probability of deviation $\tau_{reg} - \tau$ were determined (Fig. 5). Division into equal classes was assumed in agreement with criterion (1) for the upper limit of the number of classes, m. Probability, p, of obtaining a certain experimental result in a given glass of a distributive series is marked on the Y-axis on Fig. 5.

CONCLUSIONS

(a) The values of mean standard deviation, σ, and of the coefficient of variation, V, are generally lower at the reference temperature characteristic for a droplet's nucleate and film boiling than in the region of transition boiling (Fig. 4 and Table 1). The same result concerns the comparison of natural convection and transition boiling regions. It means that for established regimes of the droplet's evaporation, the scatter of evaporation times is smaller than in the transition region.

(b) In the region of minimum evaporation times, i.e. in the region of nucleate boiling crisis, the lowest values of mean standard deviation were recorded. These indicate the existence of some characteristic value of heat flux, for the given surface temperature, around which the measuring results concentrate.

(c) In the region of nucleate boiling the histogram of probability of deviation $\tau_{reg} - \tau$ has a bimodal distribution (Fig. 5(b)); for nucleate boiling crisis it becomes unimodal (Fig. 5(c)); to change into bimodal

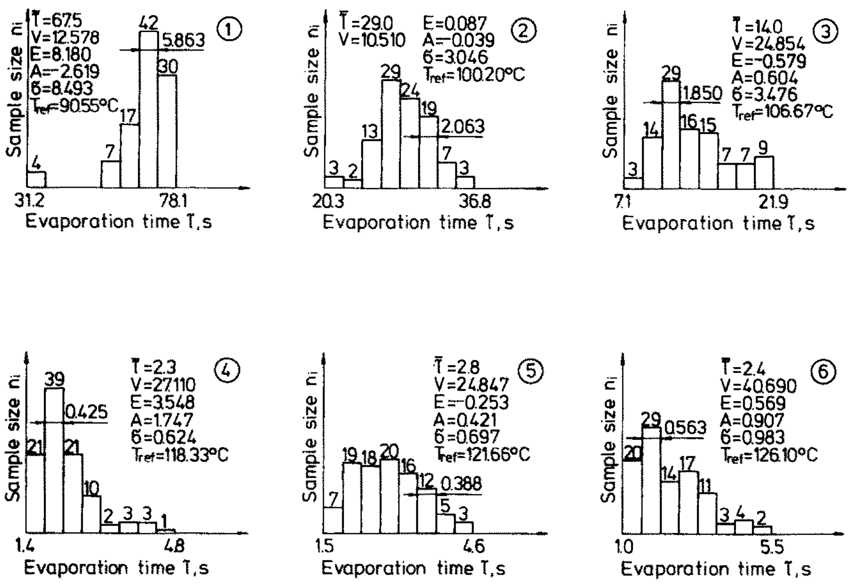

FIG. 4(a). Histograms of water droplet evaporation times—distributive series with division into eight equal classes—part I. Numbers on histograms correspond to markings on Fig. 3.

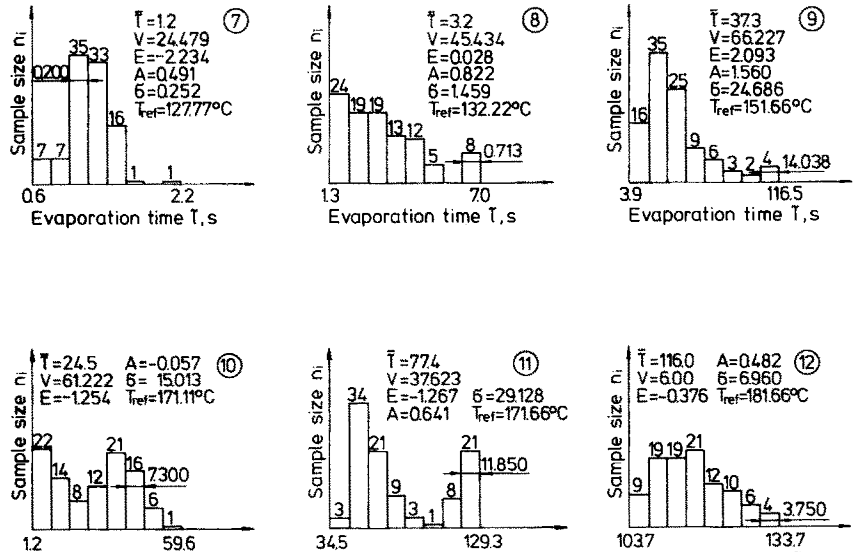

FIG. 4(b). Histograms of water droplet evaporation times—distributive series with division into eight equal classes—part II. Numbers on histograms correspond to markings on Fig. 3

for transition boiling (Fig. 5(d)). In film boiling this histogram changes into unimodal (Fig. 5(e)).

(d) The results of the statistic analysis, shown on Fig. 5, mean that, either in transition or nucleate boiling, incidental heat fluxes will concentrate around two characteristic values. For nucleate boiling (Fig. 5(b)), it can be explained by taking into account the hysteresis loop on the diagram of heat flux q as a function of temperature difference ΔT [8, 9]. This phenomenon is often called the 'zero' boiling crisis. The hysteresis of relation q vs ΔT is explained by hysteresis of activation of vapour nuclei depending on the direction of heating surface temperature changes. It is also the cause of nucleate boiling instability [8]. For nucleate boiling with forced convection, the hysteresis is explained by the possibility of the occurrence of two 'kinds' of nucleate boiling, i.e. with bubbles generated on the heating surface or with bubbles separated from this surface by a layer of superheated liquid [9].

(e) Bimodality of the probability distribution of deviations $\tau_{\text{reg}}-\tau$ for transition boiling (Fig. 5(d)) can be interpreted as concentration of the results around the values characteristic for 'transition nucleate boiling' and 'transition film boiling', according to terminology proposed by Witte and Lienhard [4]. It has been widely accepted that transition boiling is an unstable regime and the model of two transition boil-

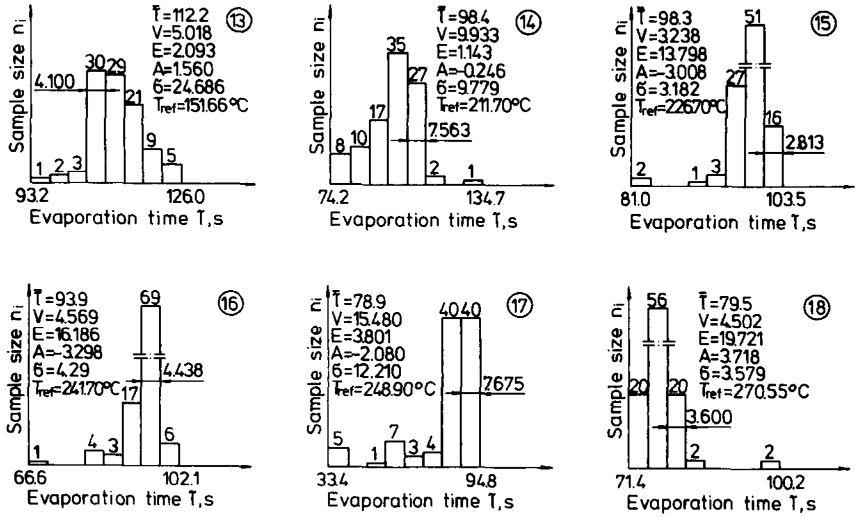

FIG. 4(c). Histograms of water droplet evaporation times—distributive series with division into eight equal classes—part III. Numbers on histograms correspond to markings on Fig. 3.

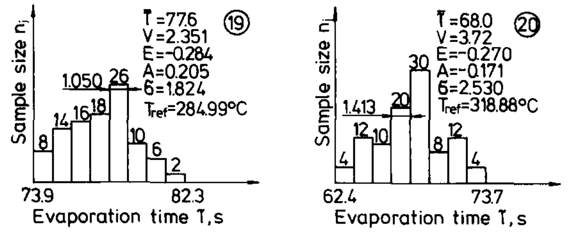

FIG. 4(d). Histograms of water droplet evaporation times—distributive series with division into eight equal classes—part IV. Numbers on histograms correspond to markings on Fig. 3.

ing curves, proposed in ref. [4], reveals the essence of this instability. Heat flux, taken from the surface in this boiling regime, oscillates between two described characteristic values. The way of conducting the experiment decides on the choice of one of them, according to ref. [4]. The possibility of the occurrence of two different boiling regimes at the same surface temperature is confirmed by numerous results of experimental studies [3, 10, 11].

(f) Unimodality of the diagram $p = p(\tau_{reg} - \tau)$ for nucleate boiling crisis (Fig. 5(c)) confirms the results obtained earlier that in the region of minimum evaporation times a certain characteristic value of heat flux exists around which the measuring results are concentrated.

(g) In agreement with numerous results of experimental studies film boiling is a stable boiling regime [12], which the unimodal probability distribution of deviations $\tau_{reg} - \tau$ confirms (Fig. 5(e)).

(h) The results (Fig. 5), make it possible to form the hypothesis of two boiling curves somewhat differently from the one by Witte and Lienhard [4]. The new hypothesis is illustrated in Fig. 6.

The envelopes of two modal values for nucleate and transition boiling are presented on Fig. 6 together with single lines corresponding to one modal value for the regions of liquid phase natural convection, nucleate boiling crisis and film boiling. The results of the statistic analysis (Fig. 5) let us assume that for

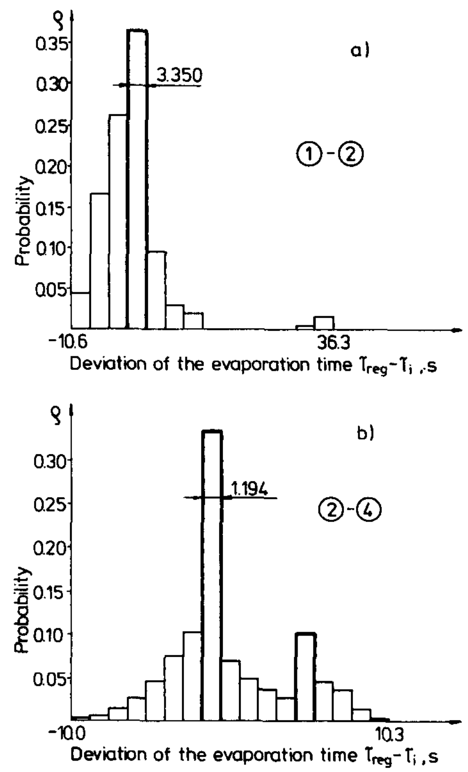

FIG. 5(a). Histograms of the probability of the deviations $\tau_{reg} - \tau$ for the characteristic intervals of the heating surface temperature—part I. The intervals are marked according to numeration of measuring points shown on Fig. 3.

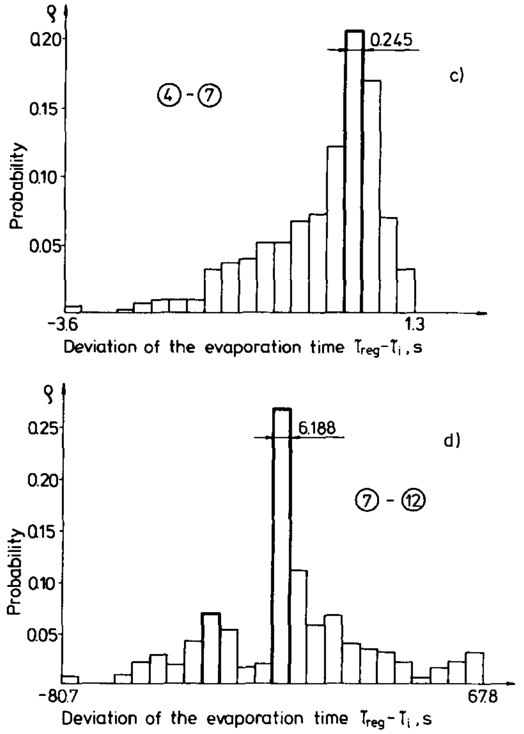

FIG. 5(b). Histograms of the probability of the deviations $\tau_{reg}-\tau$ for the characteristic intervals of the heating surface temperature—part II. The intervals are marked according to numeration of measuring points shown on Fig. 3.

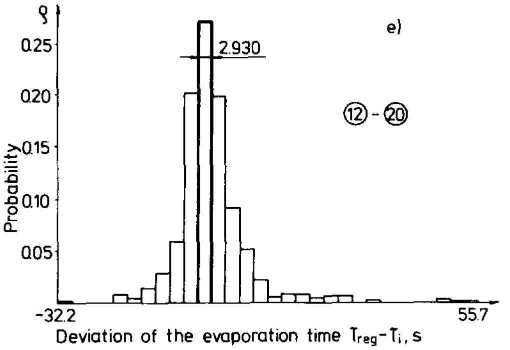

FIG. 5(c). Histograms of the probability of the deviations $\tau_{reg}-\tau$ for the characteristic intervals of the heating surface temperature—part III. The intervals are marked according to numeration of measuring points shown on Fig. 3.

the pool boiling curve, $q = q(\Delta T)$, the influence of the way of conducting the experiment on the measuring results is such as on Fig. 7. The arrows mark the direction of changes of the heating surface temperature. The experimental values of the heat flux of a boiling droplet for the regions of nucleate boiling and of transition boiling will fall accidentally on each of the two curves belonging to the two hysteresis loops.

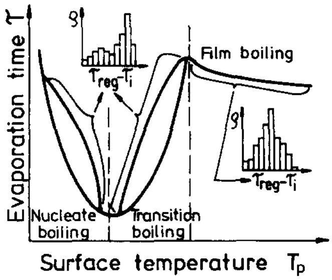

FIG. 6. Graphic interpretation of the results of the statistic analysis of water droplet evaporation times as a function of heating surface temperature.

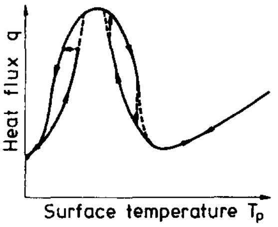

FIG. 7. Hypothesis of two boiling curves in nucleate and transition boiling.

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ANALYSE STATISTIQUE DE LA COURBE D'EBULLITION POUR UNE GOUTTELETTE

Résumé—On fait l'analyse statistique des temps d'évaporation, de gouttelettes d'eau déterminés expérimentalement. Des mesures de température de surface sont faites depuis la convection naturelle de la phase liquide jusqu'à l'ébullition en film de la goutte. Les résultats obtenus confirment l'hypothèse de transition, proposée par Witte et Lienhard. Ils conduisent à étendre ce concept à la région de l'ébullition nucléée.

EINE STATISTISCHE ANALYSE DER SIEDEKURVE EINES TROPFEN

Zusammenfassung—Eine statistische Analyse experimentell ermittelter Werte der Verdampfungszeit eines Tropfens wurde durchgeführt. Die Temperaturcharakteristik der Heizfläche wurde von der reinen einphasigen natürlichen Konvektion bis zum Filmsieden des Tropfens gemessen. Die erhaltenen Ergebnisse bestätigten die Hypothese der zwei Siedekurven im Bereich des instabilen Filmsiedens, die von Witte und Lienhard aufgestellt wurde. Die Ergebnisse erlauben auch, diesen Vorschlag auf den Bereich des Blasensiedens auszudehnen.

СТАТИСТИЧЕСКИЙ АНАЛИЗ КРИВОЙ КИПЕНИЯ ДЛЯ КАПЛИ

Аннотация—Проведен статистический анализ экспериментально определенных значений времени испарения капли. Измерялись температуры поверхности капли в режимах от естественной конвекции внутри и вплоть до пленочного кипения капли. Полученные результаты подтверждают гипотезу о двух кривых кипения в области переходного кипения, предложенную Витте и Линхардом, и дают возможность ее применения для пузырькового кипения.